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A simple modification is made to the Brouwer-Lyddane artificial satellite theory to account for the effects of atmospheric drag deceleration. The application of this drag-augmented theory to long-term station alert predictions is evaluated for several low-altitude earth satellites.

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FOREWORD

This report has been prepared to present a simple modification that can be made to the Brouwer-Lyddane artificial satellite theory to account for the effects of atmospheric drag. This analytic method for generating ephemerides for satellites has been implemented into a station alert prediction algorithm and its accuracy determined for several representative low-altitude earth satellites. The results of this study indicate that significant improvement in predicted satellite ephemerides can be made by introducing this simple drag modification into the Brouwer-Lyddane theory. This document has been reviewed and approved by R. J. Anderle.

Released by:

O. F. Braxton

O. F. BRAXTON, Head
Strategic Systems Department

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INTRODUCTION

The Brouwer-Lyddane theory is a general perturbation method that can be used to rapidly predict satellite ephemerides. This theory, however, considers only terrestrial oblateness perturbations in its formulation and takes no account of atmospheric drag accelerations. As a result, the errors induced by the drag-free Brouwer-Lyddane theory into the predicted positions and velocities of low-altitude earth satellites can be quite large and the associated ephemerides unacceptable for scheduling ground station-satellite inview periods. The inability of the Brouwer-Lyddane theory to provide sufficiently accurate long-term tracking schedules for low-altitude satellites can result in undesirable operational difficulties.

This report has been prepared to briefly discuss the effectiveness of a semi-empirical modification that can be made to the Brouwer-Lyddane theory to account for atmospheric drag in a very simple way. This modified theory can be used to predict station alert schedules for low altitude earth satellites with a much more palatable level of accuracy.

A discussion of the formulation of the semi-empirical orbit prediction model mentioned above and a review of the apparent effectiveness of this approach in diminishing drag-induced errors in the predicted satellite rise time that initiate a station inview period are given in the following sections.

FORMULATION OF THE DRAG-AUGMENTED BROUWER-LYDDANE[1,2] METHOD

The equations given below are those of the drag-augmented Brouwer-Lyddane method and consist of nothing more than the addition of time rates of change terms to the original Brouwer equations for the osculating semi-major axis (a) and eccentricity (e) and for the secular mean anomaly (ℓ''). The remaining equations for inclination (i''), argument of perigee (g) and right ascension of the ascending node (h) are unchanged. These additive terms in the equations below are enclosed in boxes for easy identification.

---

[1] Brouwer, D., "Solution of the Problem of Artificial Satellite Theory Without Drag," *The Astronomical Journal*, Vol. 64, No. 1274, 1959, pp. 378-397.

[2] Lyddane, R. H., "Small Eccentricities or Inclinations in the Brouwer Theory of Artificial Satellites," *The Astronomical Journal*, Vol. 68, No. 8, 1963, pp. 555-558.

Prior to the transcription of the drag-augmented equations, the following quantities must be defined:

$$
\left.
\begin{aligned}
\dot{a}'' &= \text{semi-major axis decay rate} \\
\dot{e}'' &= \text{eccentricity decay rate} \\
\dot{n} &= \text{time rate of change of mean motion} \\
t &= \text{time from epoch} \\
n_0 &= (\mu/a''^3)^{1/2} \\
\eta &= (1 - e''^2)^{1/2} \\
\theta &= \cos i'' \\
\gamma_2 &= 1/2\ C_{20}\ a_e^2/a''^2 \\
\gamma_2' &= \gamma_2 \eta^{-4} \\
\gamma_3' &= -\ C_{30}\ a_e^3\ a''^{-3}\ \eta^{-6} \\
\gamma_4' &= -\ 3/8\ C_{40} a_e^4\ a''^{-4}\ \eta^{-8} \\
\gamma_5' &= -\ C_{50}\ a_e^5\ a''^{-5}\ \eta^{-10} \\
\alpha &= 1 - 5\theta^2 \\
\beta &= 1 - 11\theta^2 - 40\theta^4\ \alpha^{-1} \\
\gamma &= 1 - 3\theta^2 - 8\theta^4\ \alpha^{-1} \\
\delta &= 1 - 9\theta^2 - 24\theta^4\ \alpha^{-1} \\
\lambda &= 1 - 5\theta^2 - 16\theta^4\ \alpha^{-1}\ ,
\end{aligned}
\right\} \qquad (1)
$$

where C_{j0} (j = 2, 3, 4, 5) are the zonal harmonic gravitational expansion coefficients. Then the secular terms are computed from:

$$
\begin{aligned}
\ell'' = n_0 t \Big\{ & 1 + \frac{3}{2}\gamma_2'\ \eta(3\theta^2 - 1) + \frac{3}{32}\gamma_2'^2\ \eta\ [-15 + 16\eta + 25\eta^2 \\
& + (30 - 96\eta - 90\eta^2)\theta^2 + (105 + 144\eta + 25\eta^2)\theta^4\ ] \\
& + \frac{15}{16}\gamma_4'\ \eta e''^2\ [3 - 30\theta^2 + 35\theta^4\ ] \Big\} + \ell_0'' + \boxed{\dot{n}t^2}
\end{aligned} \qquad (2)
$$

$$
\begin{aligned}
g'' = n_0 t \Big\{ & -\frac{3}{2}\gamma_2'\ \alpha + \frac{3}{32}\gamma_2'^2\ [-35 + 24\eta + 25\eta^2 \\
& + (90 - 192\eta - 126\eta^2)\theta^2 + (385 + 360\eta + 45\eta^2)\theta^4\ ] \\
& + \frac{5}{16}\gamma_4'\ [21 - 9\eta^2 + (-270 + 126\eta^2)\theta^2 + (385 - 189\eta^2)\theta^4\ ] \Big\} + g_0''\ . \qquad (3)
\end{aligned}
$$

and

$$h'' = n_0 t\left\{-3\gamma_2'\theta + \frac{3}{8}\gamma_2'^2\left[(-5+12\eta+9\eta^2)\theta + (-35-36\eta-5\eta^2)\theta^3\right]\right.$$

$$\left. + \frac{5}{4}\gamma_4'(5-3\eta^2)\theta(3-7\theta^2)\right\} + h_0''. \tag{4}$$

The long-period (dependent upon g'') terms are computed from:

$$\delta_1 e = \frac{35}{96}\frac{\gamma_5'}{\gamma_2'} e''^2 \eta^2 \lambda \sin i'' \sin^3 g'' - \frac{1}{12}\frac{e''\eta^2}{\gamma_2'}(3\gamma_2'^2\beta - 10\gamma_4'\gamma)\sin^2 g''$$

$$- \frac{35}{128}\frac{\gamma_5'}{\gamma_2'} e''^2 \eta^2 \lambda \sin i'' \sin g'' + \frac{1}{4}\frac{\eta^2}{\gamma_2'}\left[\gamma_3' + \frac{5}{16}\gamma_5'(4+3e''^2)\delta\right]$$

$$\sin i'' \sin g'' + \frac{e''\eta^2}{24\gamma_2'}[3\gamma_2^2{}'\beta - 10\gamma_4'\gamma] \tag{5}$$

$$\ell' + g' = g'' + \ell'' + \tfrac{1}{2}\left\{\frac{1}{24\gamma_2'}\right.[-3\gamma_2'^2\{2 + e''^2 - 11(2+3e''^2)\theta^2$$

$$- 40(2+5e''^2)\theta^4\alpha^{-1} - 400e''^2\theta^6\alpha^{-2}\}$$

$$+ 10\gamma_4'\{2 + e''^2 - 3(2+3e''^2)\theta^2 - 8(2+5e''^2)\theta^4\alpha^{-1} - 80e''^2\theta^6\alpha^{-2}\}]$$

$$\left. + \frac{\eta^3}{\gamma_2'}\left[\frac{\gamma_2'^2}{4}\beta - \frac{5}{6}\gamma_4'\gamma\right]\right\}\sin 2g'' + \left\{\frac{35}{384}\frac{\gamma_5'}{\gamma_2'}\eta^3 e''\lambda \sin i''\right.$$

$$+ \frac{35}{1152}\frac{\gamma_5'}{\gamma_2'}\left[\lambda\left\{-e''(3+2e''^2)\sin i'' + \frac{e''^3\theta^2}{\sin i''}\right\}\right.$$

$$\left.\left. + 2e''^3\theta^2 \sin i''\{5 + 32\theta^2\alpha^{-1} + 80\theta^4\alpha^{-2}\}\right]\right\}\cos 3g''$$

$$+ \left\{-\frac{\gamma_3 e''\theta^2}{4\gamma_2'\sin i''} + \frac{5}{64}\frac{\gamma_5'}{\gamma_2'}\left[-e''\frac{\theta^2}{\sin i''}(4+3e''^2) + e''^2 \sin i''\right.\right.$$

$$(26 + 9e''^2)\Big]\delta - \frac{15}{32}\frac{\gamma_5'}{\gamma_2'} e''\theta^2 \sin i'' (4 + 3e''^2)$$

$$(3 + 16\theta^2\alpha^{-1} + 40\theta^4\alpha^{-2}) + 1/4 \frac{\gamma_3'}{\gamma_2'} \sin i''.$$

$$\left(\frac{e''}{1+\eta^3}\right)[3 - e''^2 (3 - e''^2)] + \frac{5}{64}\frac{\gamma_5'}{\gamma_2'} \eta^2 \delta$$

$$\left[\frac{e''(-32 + 81e''^4)}{4 + 3e''^2 + \eta(4 + 9e''^2)}\right] \sin i''\Big\}\cos g'' \qquad (6)$$

and

$$h' = h'' + \frac{35\gamma_5' e''^3\theta}{144\gamma_2'}\left\{\frac{1}{2}\lambda \sin^{-1} i'' + \sin i'' [5 + 32\theta^2\alpha^{-1} + 80\theta^4\alpha^{-2}]\right\}$$

$$\sin^2 g'' \cos g'' + \frac{e''^2\theta}{12\gamma_2'}\left\{- 3\gamma_2'^2 [11 + 80\theta^2\alpha^{-1} + 200\theta^4\alpha^{-2}]\right.$$

$$\left. + 10\gamma_4' [3 + 16\theta^2\alpha^{-1} + 40\theta^4\alpha^{-2}]\right\} \sin g'' \cos g''$$

$$+\left\{- \frac{35\gamma_5'}{576\gamma_2'} e''^3\theta\left[\frac{1}{2}\lambda \sin^{-1} i'' + \sin i'' (5 + 32\theta^2\alpha^{-1} + 80\theta^4\alpha^{-2})\right]\right.$$

$$+ \frac{e''\theta}{4\gamma_2' \sin i''}\left[\gamma_3' + \frac{5}{16}\gamma_5' (4 + 3e''^2)\delta + \frac{15}{8}\gamma_5' (4 + 3e''^2)\right.$$

$$\left.\left.(3 + 16\theta^2\alpha^{-1} + 40\theta^4\alpha^{-2}) \sin^2 i''\right]\right\}\cos g'' . \qquad (7)$$

The short periodics (dependent upon E', f', ℓ'') are computed from:

$$a = \boxed{\dot{a}''t} + a'' - a''\frac{\gamma_2}{\eta^3}(3\theta^2 - 1) + \left[\frac{a''\gamma_2}{(1 - e'' \cos E')^3}\right]$$

$$[3\theta^2 - 1 + 3 \sin^2 i'' \cos(2g'' + 2f')] \qquad (8)$$

$$e = e'' + \boxed{\dot{e}''t} + \delta_1 e + \frac{\eta^2\gamma_2}{2}\left\{\frac{3\theta^2-1}{\eta^6}\left[\frac{e''}{1+\eta^3}\left\{3-e''^2\,(3-e''^2)\right\}\right.\right.$$

$$\left.+\left\{3+e''\cos f'\cdot(3+e''\cos f')\right\}\cos f'\right]+\frac{3(1-\theta^2)}{\eta^6}$$

$$\left.[e''+\left\{3+e''\cos f'\,(3+e''\cos f')\right\}\cos f']\cos(2f'+2g'')\right\}$$

$$-\frac{\eta^2\gamma_2'}{2}(1-\theta^2)[3\cos(2g''+f')+\cos(2g''+3f')] \tag{9}$$

$$i = i'' - \frac{e''\theta}{\eta^2\sin i''}\delta_1 e + e''\gamma_2'\theta\,\sin i''\sin f'\sin(2f'+2g'')$$

$$+2e''\gamma_2'\theta\sin i''\cdot\cos f'\cos(2f'+2g'')+\frac{3}{2}\gamma_2'\theta\sin i''\cos(2f'+2g'') \tag{10}$$

$$g+\ell = g'+\ell'+\frac{\gamma_2'}{4}\left\{-6\alpha(f'-\ell''+e''\sin f')+(3-5\theta^2)\right.$$

$$\left.[3\sin(2f'+2g'')+3e''\sin(2g''+f')+e''\sin(2g''+3f')]\right\}$$

$$+\frac{e''\eta^2\gamma_2'}{4(1+\eta)}\left\{2(3\theta^2-1)(\sigma+1)\sin f'+3(1-\theta^2)\right.$$

$$\left.[(1-\sigma)\sin(2g''+f')+(\sigma+1/3)\sin(2g''+3f')]\right\} \tag{11}$$

$$h = h'+[2e''\gamma_2'\,\theta\cos f'+\frac{3}{2}\gamma_2'\theta\,]\sin(2g''+2f')$$

$$-e''\gamma_2'\theta\sin f'\cos(2f'+2g'')-3\gamma_2'\theta(f'-\ell''+e''\sin f') \tag{12}$$

and

$$e\delta\ell = \frac{1}{2}\frac{e''\eta^3}{\gamma_2'}\left\{\frac{1}{4}\gamma_2'\beta-\frac{5}{6}\gamma_4'\gamma\right\}\sin 2g''$$

$$- \left\{ \frac{1}{4} \frac{\gamma_3'}{\gamma_2'} \eta^3 \sin i'' + \frac{5}{64} \frac{\gamma_5'}{\gamma_2'} \eta^3 \sin i'' (4 + 9e''^2) \delta \right\}.$$

$$\cos g'' + \frac{35}{384} \frac{\gamma_5'}{\gamma_2'} \eta^3 e''^2 \lambda \sin i'' \cos 3g''$$

$$- \frac{1}{4} \gamma_2' \eta^3 \left\{ 2(3\theta^2 - 1)(\sigma + 1) \sin f' + 3(1 - \theta^2)(1 - \sigma) \sin(2g'' + f') \right.$$

$$\left. + \left(\sigma + \frac{1}{3}\right) \sin(2g'' + 3f') \right\}, \tag{13}$$

where

$$\sigma = \left(\frac{\eta}{1 - e'' \cos E'} \right)^2 + \left(\frac{1}{1 - e'' \cos E'} \right), \tag{14}$$

the eccentric anomaly E' is obtained from a Newton-Raphson iteration upon the Kepler equation

$$E' - e'' \sin E' = \ell'' \ , \tag{15}$$

and the true anomaly f' is found from

$$\sin f' = \frac{\eta \sin E'}{1 - e'' \cos E'},$$

$$\cos f' = \frac{\cos E' - e''}{1 - e'' \cos E'} \ . \tag{16}$$

The final osculating values for a, i, and h are computed from equations (8), (10), and (12), respectively. Equations (2), (9), (11), and (13) are used to calculate final osculating values for ℓ, g, and e from the following relations:

$$A = e \cos \ell'' - e \delta \ell \sin \ell'' \ , \tag{17}$$

$$B = e \sin\ell'' + e\delta\ell \cos\ell'' \quad , \tag{18}$$

$$\ell = \tan^{-1}(B/A) \quad , \tag{19}$$

$$g = (\ell + g) - \ell \quad , \tag{20}$$

and

$$e = (A^2 + B^2)^{1/2} \quad . \tag{21}$$

All of the input data required to generate the osculating orbital elements from the above-augmented equations; i.e., a'', $\dot{a}''$, e'' $\dot{e}''$, i'', ℓ_0'', $\dot{n}$, g_0'', and h_0'', can be obtained directly from the NAVSPASUR five-card element format data.[3] However, the semi-major axis and associated decay rate provided there are the Kaula semi-major axis (a_k) and decay rate ($\dot{a}_k$) expressed in earth radii. The Kaula semi-major axis can be converted to the desired linear units, when one uses the relation:

$$a'' = a_k\, a_e \left(\frac{1 + 2X}{1 - X}\right)^{2/3} \quad , \tag{22}$$

where a_e is the earth's semi-major axis,

$$X = \frac{3J_2(1 - 3/2 \sin^2 i'')}{4\, a_k^2\, (1 - e''^2)^{3/2}} \quad , \tag{23}$$

and J_2 = [illegible] zonal harmonic gravitational constant.

Similarly, the desired decay rate $\dot{a}''$ can be obtained from $\dot{a}_k$ by taking the time derivative of equation (22). This gives

$$\dot{a}'' = \dot{a}_k\, a_e \left(\frac{1 + 2X}{1 - X}\right)^{2/3} + 2\, a_k\, a_e\, \dot{X}\, [(1 + 2X)(1 - X)^5]^{-1/3} \quad , \tag{24}$$

[3]McCandless, A. R., "NAVSPASUR, SPADATS Type, 5-Card Element Format; description of," memorandum V221:EAT:jmd, 3163, 20 September 1967.

where

$$\dot{X} = X\left[3\left(\frac{e''}{1 - e''^2}\right)\dot{e}'' - 2\left(\frac{\dot{a}_k}{a_k}\right)\right]. \tag{25}$$

In the last expression it has been assumed that

$$(\dot{i}'') = 0. \tag{26}$$

EVALUATION METHOD

In order to evaluate the utility of the drag-augmentation method, five-card element sets were obtained from NAVSPASUR for two 92-min. period satellites at 7-day intervals for the timespan from 9 January 1981 through 24 April 1982. During this timespan, these two satellites had perigee altitudes which remained in a 160-nmi to 180-nmi altitude box.

These data were used to generate 30-day predicted station alert schedules every 7 days during the timespan for a single tracking station. The satellite rise times generated near the epochs of the 7-day element sets were assumed to be the true rise times, t_{R_T}. These were compared with the associated predicted rise times, t_{R_P}, generated for 30 days at 7-day epoch intervals. Orbit adjust schedules, obtained for these two satellites for the span of interest, were used to eliminate those values of t_{R_P} which were invalidated due to orbit adjusts occurring at some time during the prediction interval. The remaining valid t_{R_P} were differenced with the associated t_{R_T} and their absolute values averaged as a function of prediction interval.

RESULTS AND CONCLUSIONS

The results of the averaging procedure described in the previous section are shown in Figure 1 along with the associated 2σ confidence envelope. Also included on this figure is a plot of the errors obtained from a typical drag-free Brouwer-Lyddane prediction for one of the two satellites discussed previously when its perigee altitude was ~178 nmi and its argument of perigee was on the sunlit side of the earth. It is apparent from these three curves that the inclusion of the simple drag decay terms in the Brouwer-Lyddane theory has dramatically improved the long-term quality of the predicted satellite rise times.

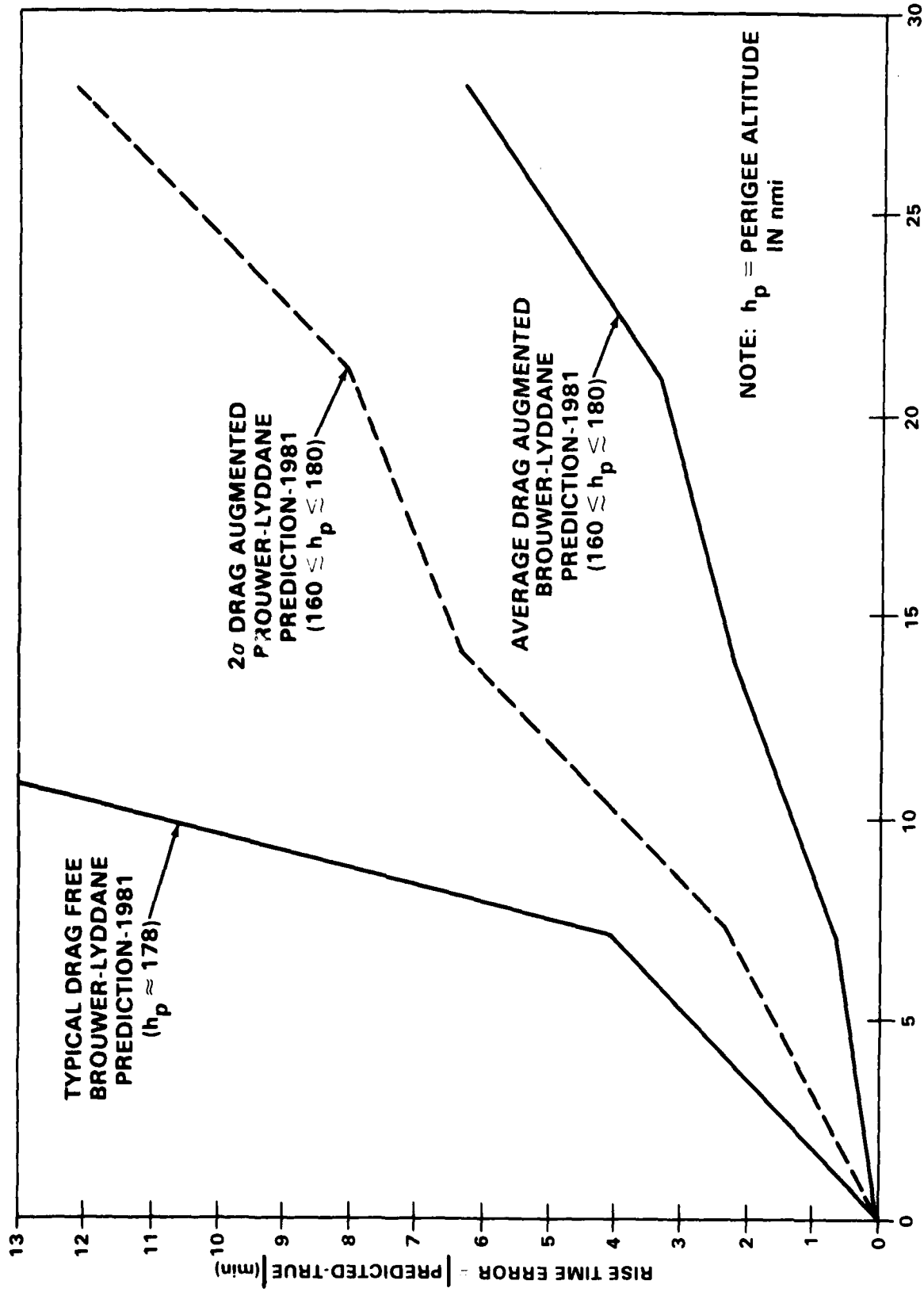

FIGURE 1. PREDICTION ERROR VS. PREDICTION INTERVAL

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